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Distributed Storage Management of Evolving Files in Delay Tolerant Ad Hoc Networks

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Abstract: This work focuses on a class of distributed storage systems whose content may evolve over time. Each component or node of the storage system is mobile and the set of all nodes forms a delay tolerant (ad hoc) network (DTN). The goal of the paper is to study efficient ways for distributing evolving files within DTNs and for managing dynamically their content. We specify to dynamic files where not only the latest version is useful but also previous ones; we restrict however to files where a file has no use if another more recent version is available. There are $N+1$ mobile nodes including a *single* source which at some points in time makes available a new version of a *single* file F. We consider both the cases when (a) nodes do not cooperate and (b) nodes cooperate. In case (a) only the source may transmit a copy of the latest version of F to a node that it meets, while in case (b) any node may transmit a copy of F to a node that it meets. A file management policy is a set of rules specifying when a node may send a copy of F to a node that it meets. The objective is to find file management policies which maximize some system utility functions under a constraint on the resource consumption. Both myopic (*static*) and state-dependent (*dynamic*) policies are considered, where the state of a node is the age of the copy of F it carries. Scenario (a) is studied under the assumption that the source updates F at discrete times $t = 0, 1, \ldots$. During a slot $[t, t+1)$ the source meets any node with a fixed probability

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q. We find the optimal static (resp. dynamic) policy which maximizes a general utility function under a constraint on the number of transmissions within a slot. In particular, we show the existence of a threshold dynamic policy. In scenario (b) F is updated at random points in time. We will consider both the case where at any time nodes know the age of the file they carry and the case where they only know the time at which the file they carry was created by the source. Under Markovian assumptions regarding nodes mobility and update frequency of F, we study the stability of the system (aging of the nodes) and derive an (approximate) optimal static policy. We then revisit scenario (a) when the source does not know the number of nodes and the probability that the source meets a node in a slot, and we derive a stochastic approximation algorithm which we show to converge to the optimal static policy found in the complete information setting. Numerical results illustrate the respective performance of optimal static and dynamic policies as well as the benefit of node cooperation.

Key-words: Evolving files; Storage systems; Delay-tolerant (ad hoc) networks; Performance evaluation; Optimization.

Gestion du Stockage Distribué de Fichiers Evolutifs dans les Réseaux Ad Hoc Tolérants les Délais

Résumé : Nous nous intéressons à une classe de systèmes de stockage distribué dont le contenu évolue au cours du temps. On parle de fichiers (ou documents) évolutifs (ou dynamiques). Chaque composante ou nœud du système de stockage est mobile et l'ensemble des nœuds forme un réseau ad hoc tolérant les délais. L'objectif de cet article est l'étude de politiques efficaces de distribution et de gestion de fichiers dynamiques dans les réseaux ad hoc. Nous considérons des fichiers évolutifs dont non seulement la dernière version est utile mais aussi les versions antérieures; toutefois, nous nous restreignons à des documents pour lesquels il est suffisant de conserver la version la plus récente. Il y a $N+1$ noeuds mobiles y compris un nœud source qui à certains instants génère une nouvelle version d'un fichier unique, appelé F. Deux cas sont analysés selon que (a) les nœuds ne coopérent pas ou (b) coopérent à la distribution du fichier F. Dans le cas (a) seule la source est autorisée à transmettre une copie de la version courante de F à un nœud qu'elle rencontre alors que dans le cas (b) chaque nœud qui possède une version de F à la possibilité d'en transmettre une copie à un autre nœud qu'il rencontre. Une politique de gestion des fichiers est un ensemble de règles spécifiant si lors d'une rencontre entre deux nœuds l'un des deux peut transmettre une copie de F à l'autre. Le but est d'identifier des politiques de gestion qui maximisent une fonction d'utilité du système sous une contrainte portant sur la consommation des ressources. A la fois des politiques statiques et dynamiques sont considérées, où l'état d'un nœud est l'âge de la version de F qu'il possède. Le scénario (a) ci-dessus est analysé sous l'hypothèse où les mises à jour de F surviennent à des instants discrets $t = 0, 1, \ldots$. Durant chaque intervalle $[t, t+1)$ la source rencontre chacun des nœuds avec une probabilité fixée q. Nous calculons la politique qui maximise une fonction d'utilité de portée générale, la contrainte portant sur le nombre moyen de transmissions dans $[t, t+1)$. En particulier, nous montrons l'existence d'une politique dynamique optimale de type seuil. Dans le scénario (b) le fichier F est mis à jour par la source à des instants aléatoires. Nous considérons à la fois le cas où, à chaque instant, les nœuds connaissent l'âge du fichier qu'ils possèdent et le cas où ils ne connaissent que la date de création de ce fichier. Sous des hypothèses markoviennes quant à la mobilité des nœuds et la fréquence de mise à jour de F, nous établissons la condition de stabilité du système (processus de vieillissement des versions de F à chaque nœud) et proposons un algorithme approché pour le calcul d'une politique statique optimale. Nous revenons ensuite au scénario (a) en supposant que la source ne possède qu'une information partielle du système (elle ne connait ni N ni q) et développons un algorithme d'approximation stochastique convergeant vers la politique statique optimale obtenue dans le cas d'information totale. Des résultats numériques illustrent les performances respectives des politiques statiques et dynamiques ainsi que le bénéfice qu'apporte la coopération inter-nœuds.

Mots-clés : Fichiers évolutifs; Systèmes de stockage; Réseaux ad hoc tolérants les délais; Evaluation des performances; Optimisation.

1 Introduction

Much work has been devoted for the study of Delay Tolerant Networks (DTNs). Most of the work on protocol design has focused on the use of mobility in order to reach one or more disconnected destinations. The protocols are based on distribution of the file to relay nodes so as to increase the successful delivery probability [2, 3, 4, 10, 12].

In such applications, the DTN becomes a distributed storage system that contains copies of a file that is being transmitted. In this paper we focus on a special type of file that we call "dynamic file" or "evolving file". By that we mean a file whose content may evolve and change from time to time. One (or various) sources wish to make a file available to mobile nodes, and to send updates from time to time. Some examples are:

- a source has a file containing update information such as weather forecast or news headlines. The file changes incrementally from time to time with new information updates;
- a source wishes to make backups of some directories and to store them at another nodes in order to increase the reliability;
- some software updates or patches may be distributed regularly.

Several formats of dynamic files have been standardized:

- the RSS ("Real Simple Syndication" [5]) family of Web feed formats used to publish frequently updated content such as blog entries, news headlines, and podcasts in a standardized format. Updates can originate from various sources;
- another format called the "Atom Syndication Format" has been adopted as IETF Proposed Standard RFC 4287.

We specify to dynamic files where not only the latest version is useful but also previous ones; we restrict however to files where a file has no use if another more recent version is available. For example, consider an evolving file containing the weather forecast for seven consecutive days. If a user needs the weather forecast for the next day then any version of the file from the six last days is useful. The more recent the file is, the more accurate the requested information is. Furthermore, having access to a given file makes all previous files irrelevant to the user.

The goal of our paper is to study efficient ways for distributing evolving files within DTNs and for managing dynamically their content. The obvious way to provide the most up-to-date information is to use epidemic routing (e.g. see [12]) for each new version of F. This however consumes a lot of network resources.

We start with a general description of the model. More details will be given in the subsequent sections. There are $N+1$ mobile nodes including one source node. From now on a *node* designates any mobile node other than the source. At some time epochs the source creates an updated version of a file F. When the source meets a node it may decide to transmit to this node a copy of F. Similarly, when two nodes meet the node which carries the more recent version of F may transmit a copy of this version to the other node. When a node receives a more recent version of F than the one it was carrying (if any) it deletes at once the oldest version of F.

The setting in which only the source may transmit (a copy of) F to another node is called the *non-cooperative* setting, while in the *cooperative* setting any mobile node may transmit to any other node. We assume that transmissions are always successful.

There is a utility $U(k)$ associated with a node in state k, where the state of a node is defined as the age of the copy of F, if any, this node carries. A *file management policy*, or simply a policy, is a set of rules specifying whether the source and a node, or two nodes, should communicate whenever they meet. A policy is *static* (resp. *dynamic*) if the decision to transmit does not (resp. does) depend on the state of the mobile nodes. The objective is to find a file management policy that maximizes the expected system utility given a constraint on the expected number of communications taking place in a slot.

Section 2 addresses the non-cooperative setting. Time is slotted and there is a fixed probability q that any pair of mobile nodes meets in a slot. At the beginning of each slot the source creates a new version of F, so that each node carrying a copy of F knows that its state has increased by one unit. A copy of F reaching age $K+1$ ($K<\infty$) is immediately deleted. We find the optimal static policy (Proposition 1) and show that there is an optimal dynamic policy of a threshold type (Proposition 2) which we fully characterize (Proposition 3). The performance of the optimal static and dynamic policies are compared (Figures 1-4) for two different utility functions ($U(k)=1$ and $U(k)=1/k$).

Section 3 investigates the cooperative setting. We develop a continuous-time model in which mobile nodes meet at random times and file F is updated by the source also at random times. We consider both the situation when (i) nodes know the age of the file they carry (Section 3.1) and the case (ii) where they only know the date at which the file was created by the source (Section 3.2). In the situation where a node only deletes a file if it replaces it by a more recent version (i.e. $K=\infty$), we study the stability of the network, in a Markovian framework, for both settings (i) (Proposition 4) and (ii) (Proposition 5) above. Here, stability refers to the state or age of each node being almost surely finite. Under the more restrictive assumptions where node meeting times and update times are modeled by independent Poisson processes, we derive a "mean-field like" approximation for the expected number of nodes in state $k\geq 1$ in the case where a static policy is enforced. We then use this result to quantify in Figures 8-9 the benefit of having nodes to cooperate.

The deployment of optimal policies derived in Sections 2-3 requires that the source has a complete information on the network (node mobility, number of nodes). In Section 4 we release this assumption. We focus on the noncooperative setting and restrict to static policies, and we assume that the source does not know the number of nodes N and does not know the meeting probability q. By using the theory of stochastic approximations, we construct an algorithm which converges to the optimal static policy found in Section 2. Section 5 concludes the paper.

Remark on the notation: by convention $\sum_{k=i}^{j} \cdot = 0$ and $\prod_{k=i}^{j} \cdot = 1$ if $i > j$. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers.

2 Non-cooperative nodes

In this section we consider the scenario where nodes do not cooperate and may only receive file F from the source. Nodes are labeled $1, 2, \ldots, N$. At times $t = 0, 1, \ldots$ the source creates a new version of file F. In the following, a slot denotes any time-period $[t, t+1)$, $t \geq 0$, and slot t stands for the time-period $[t, t+1)$. There is a probability $q(i) > 0$ that node $i = 1, \ldots, N$ meets the source in a slot. We define the *meeting times* between the source and a node as the successive slots at which they meet. The meeting times of each node which the source form a sequence of independent and identically distributed (iid) random variables (rvs) and all meeting time processes are assumed to be mutually independent. For sake of simplicity, we assume that all transmissions between the source and the nodes initialized in a slot are completed by the end of this slot. This implies that the transmission time of F is small w.r.t. the duration of a slot.

When a node receives an updated version of F it deletes at once the previous version of F it was carrying, if any. We define the age of a version of F as the number of slots that have elapsed since this version was generated by the source. We assume that a version of age $K + 1$ or more is useless and that a node deletes at once a file that has reached age $K + 1$. Therefore, the age of a version of F varies between 1 (the version was generated in the current slot) and K (the version was generated $K - 1$ slots ago). We further assume that $K < \infty$ (see Remark 2.1).

The state of a node is defined as the age of the version of F it carries, if any. A node is in state 0 if it does not carry any version of F. A node in state K at the end of a slot switches to state 0 at the beginning of the next slot.

When the source meets node i, with probability $a_k(i)$ it transmits to it the newest version of F if that node is in state k ($k = 0, 1, \ldots, K$). We assume that the transmission is always successful. The decision by the source to transmit to a node is independent of all past decisions made by the source and is also independent of all meeting time processes. Introduce $p_k(i) := q(i)a_k(i)$ the probability that node i in state k receives the newest version of F in a slot. Define $p_k^c(i) := 1 - p_k(i)$. At equilibrium, let $\pi_k(i)$ be the probability that node i is in state k at the end of a slot, and let $\overline{X}_k$ be the average number of nodes in state

k at the end of a slot. We have

$$\overline{X}_k = \sum_{i=1}^{N} \pi_k(i), \quad k = 0, 1, \ldots, K, \tag{1}$$

with $\sum_{k=0}^{K} \overline{X}_k = N$.

For each $i = 1, \ldots, N$, the probabilities $\{\pi_k(i)\}_{k=0}^{K}$ satisfy the Chapman-Kolmogorov equations

$$\begin{aligned} \pi_0(i) &= \pi_0(i) p_0^c(i) + \pi_K(i) p_K^c(i) && (2)\\ \pi_k(i) &= \pi_{k-1}(i) p_{k-1}^c(i), \quad k = 2, \ldots, K, && (3)\\ 1 &= \sum_{k=0}^{K} \pi_k(i). && (4) \end{aligned}$$

There is one additional equilibrium equation given by $\pi_1(i) = \sum_{k=0}^{K} \pi_k(i) p_k(i)$ which we will not consider since it can be derived by summing up equations (2)-(3). Equations (2)-(4) define a linear system of $K+1$ equations and $K+1$ unknowns.

From now on we will assume that $p_0(i) > 0$ (i.e. $a_0(i) > 0$ since we have assumed that $q(i) > 0$) for all i as otherwise the solution to (2)-(4) may not be unique. The non-uniqueness of the solution corresponds to situations where the steady-state of node i will depend upon its initial state (e.g. take $p_1(i) = 1$ and $p_k(i) = 0$ for $k \neq 1$), a degenerated situation that can easily be handled and that we will not consider from now on. Solving for (2)-(4) gives

$$\pi_0(i) = \frac{\prod_{k=1}^{K} p_k^c(i)}{D_i}, \quad \pi_k(i) = \frac{p_0(i) \prod_{l=1}^{k-1} p_l^c(i)}{D_i} \tag{5}$$

for $k = 1, \ldots, K$, $i = 1, \ldots, N$, with

$$D_i := p_0(i) \sum_{k=1}^{K} \prod_{l=1}^{k-1} p_l^c(i) + \prod_{k=1}^{K} p_k^c(i). \tag{6}$$

Hence, by (1),

$$\overline{X}_0 = \sum_{i=1}^{N} \frac{\prod_{k=1}^{K} p_k^c(i)}{D_i}, \quad \overline{X}_k = \sum_{i=1}^{N} \frac{p_0(i) \prod_{l=1}^{k-1} p_l^c(i)}{D_i} \tag{7}$$

for $k = 1, \ldots, K$.

In the particular case where $p_k(i) = p_k$ for all i, k then

$$\overline{X}_0 = \frac{N \prod_{k=1}^{K} (1 - p_k)}{D}, \quad \overline{X}_k = \frac{N p_0 \prod_{l=1}^{k-1} (1 - p_l)}{D} \tag{8}$$

for $k = 1, \ldots, K$, where

$$D := p_0 \sum_{k=1}^{K} \prod_{l=1}^{k-1} (1 - p_l) + \prod_{k=1}^{K} (1 - p_k). \tag{9}$$

If we further assume that $p_k = p$ for $k = 0, 1, \ldots, K$ then

$$\overline{X}_0 = N(1-p)^K, \quad \overline{X}_k = Np(1-p)^{k-1},\ k = 1, \ldots, K. \tag{10}$$

Remark 2.1 **($K = \infty$)** *Formulas (7) hold if $K = \infty$ (i.e. nodes never delete the file they carry unless they receive a new version from the source) provided that D_i in (6) is finite for every i as $K \uparrow \infty$. This is so if $\lim_{k\uparrow\infty} p_k(i) > 0$ for $i = 1, \ldots, N$ (Hint: apply d'Alembert's criterion to the series $\sum_{k\geq 1} \prod_{l=1}^{k-1} p_l^c(i))$). Note from (7) that $\overline{X}_0 = 0$ if $K = \infty$.*

Remark 2.2 (Intermittently available nodes) *The situation where nodes are intermittently available can be handled by replacing $p_k(i)$ by $r(i)p_k(i)$ with $r(i)$ the probability that node i is available in a slot.*

2.1 Performance metrics

There are several performance metrics of interest which can be derived from (8). One of these is the expected number copies of F given by

$$\overline{X} = \sum_{k=1}^{K} \overline{X}_k = N - \overline{X}_0. \tag{11}$$

Another one is the expected age of the copies given by $(1/N)\sum_{k=1}^{K} k\overline{X}_k$. Of particular interest is to evaluate the power consumption. Since the power consumption, denoted as Q, is proportional to the expected number of transmissions during a slot, we will define it as

$$Q = \gamma \overline{X}_1. \tag{12}$$

Without loss of generality we assume from now on that $\gamma = 1$.

2.2 Energy efficient file management policies

Until the end of Section 2 we will assume that nodes are homogeneous in the sense that $q(i) := q$ and $a_k(i) := a_k$ for all i, k with $q > 0$. We assume that $a_0 > 0$. We define $p_k := qa_k$ so that $\mathbf{p} = q\,\mathbf{a}$ with $\mathbf{a} = (a_0, \ldots, a_K)$. In this setting $\{\overline{X}_k\}_{k=0}^{K}$ are given in (8).

In this framework the expected age of the copies and the power consumption introduced in Section 2.1 are denoted by $Age(\mathbf{p})$ and $Q(\mathbf{P})$, respectively, so as to stress their dependency on the vector $\mathbf{p}$. More precisely,

$$Age(\mathbf{p}) = \frac{1}{N}\sum_{k=1}^{K} k\overline{X}_k, \tag{13}$$

$$Q(\mathbf{p}) = \overline{X}_1. \tag{14}$$

A file management policy is any decision vector $\mathbf{a} = (a_0, \ldots, a_K) \in (0,1] \times [0,1]^K$, where we recall that a_k is the (conditional) probability that the source transmits F to a node in state k when it meets such a node. An equivalent definition of a file management policy is any vector $\mathbf{p} = (p_0, \ldots, p_K) \in (0,q] \times [0,q]^K$ since $\mathbf{p} = q\mathbf{a}$. Unless otherwise mentioned we will work with the latter definition.

Our objective is to find an optimal file management policy $\mathbf{p}$ which maximizes the system utility given a power consumption constraint. More precisely, let $U(k)$ be the utility for having a file of age k in the system. We assume that the mapping $U : \{0, 1, \ldots, K\} \to \mathbb{R}^+$ is non-increasing. Without loss of generality we assume $U(0) = 0$. The system utility is defined as

$$C(\mathbf{p}) = \sum_{k=1}^{K} \overline{X}_k U(k). \tag{15}$$

If $U(k) = 1$ for all $k > 0$ then $C(\mathbf{p}) = \overline{X}$, given in (11). We will assume that U is not identically zero as otherwise the system utility is always zero.

The optimization problem is the following:

P: Maximize $C(\mathbf{p})$ over the set $(0,q] \times [0,q]^K$ given $Q(\mathbf{p}) \le V$, where V is a positive constant.

We will solve **P** in two different settings: the *static* setting where management policies are restricted to policies of the form $\mathbf{p} = (p, \ldots, p)$ with $p \in (0,q]$, and the *dynamic* setting where the optimization is made over all vectors $\mathbf{p} = (p_0, \ldots, p_K) \in (0,q] \times [0,q]^K$.

2.2.1 Static optimal policy

In the static setting, problem **P** becomes (see (10)):

P: Maximize $C(p) := Np\sum_{k=1}^{K}(1-p)^{k-1}U(k)$ over $p \in (0,q]$ given that $Np \le V$.

The following result holds:

Proposition 1 (Optimal static policy)

If $Nq \le V$ then $p^\star = q$ is the optimal solution; otherwise $p^\star = V/N$ is the optimal solution or, equivalently, $p^\star = \min(q, V/N)$.

It is enough to show that the mapping $p \to C(p)$ is strictly increasing in $(0, q)$. Define $U(K+1) = 0$. We have from (15) and (10)

$$\begin{aligned} C(p) &= \sum_{k=1}^{K} (U(k) - U(k+1)) \sum_{l=1}^{k} \overline{X}_l \\ &= N \sum_{k=1}^{K} (U(k) - U(k+1))(1 - (1-p)^k), \end{aligned}$$

Hence, $dC(p)/dp = N \sum_{k=1}^{K} (U(k) - U(k+1)) k (1-p)^{k-1} > 0$ for $p \in (0, q)$, since U is non-increasing and not identically zero (which necessarily implies that $U(K) > 0$). ∎

2.2.2 Dynamic optimal policy

Let us introduce the new decision variables $x_k = 1 - p_k$ for $k = 1, \ldots, K$ and $x_K = (1 - p_K)/p_0$. Note that $1 - q \leq x_k \leq 1$ for $k = 1, \ldots, K$ and $x_K \geq (1-q)/q$ with equality if and only if $p_0 = p_K = q$. Let $\mathbf{x} = (x_1, \ldots, x_K)$. Introduce the set

$$\mathbf{E} = \left\{ \mathbf{x} \, : \, \mathbf{x} \in [1-q, 1]^{K-1} \times [(1-q)/q, \infty) \right\}.$$

Any vector $\mathbf{x} \in \mathbf{E}$ is called a *policy.* Define the mappings

$$F(\mathbf{x}) = \sum_{k=1}^{K} U(k) \prod_{l=1}^{k-1} x_l, \quad G(\mathbf{x}) = \sum_{k=1}^{K+1} \prod_{l=1}^{k-1} x_l$$

and let $H(\mathbf{x}) := F(\mathbf{x})/G(\mathbf{x})$. Note that $F(\mathbf{x})$ does not depend on the variable x_K. From (9) $D = p_0 G(\mathbf{x})$, and so by (8)

$$C(\mathbf{p}) = N H(\mathbf{x}) \quad \text{and} \quad Q(\mathbf{p}) = \frac{N}{G(\mathbf{x})}.$$

In this new notation problem $\mathbf{P}$ becomes $\max_{\mathbf{x} \in \mathbf{E}} H(\mathbf{x})$ subject to the constraint $G(\mathbf{x}) \geq C$, with $C := N/V$.

An admissible policy is any policy such that $G(\mathbf{x}) \geq C$.

Definition 2.1 (Threshold policy)

A policy $\mathbf{x} = (x_1, \ldots, x_K) \in \mathbf{E}$ is a threshold policy if either $x_k = 1$ or $x_{k+1} = 1 - q$ for $k = 1, \ldots, K-2$ and if either $x_{K-1} = 1$ or $x_K = (1-q)/q$.

Any threshold policy $\mathbf{x} = (x_1, \ldots, x_K)$ is such that $x_1 \geq \ldots \geq x_{K-1}$. More precisely, it is easily seen that a threshold policy if either of **Type I** or of **Type II** with

Type I: for $k = 1, \dots, K$

$$\mathbf{x}_k(\alpha) := (1, \dots, 1, \alpha, 1-q, \dots, 1-q, (1-q)/q) \tag{16}$$

where $1 - q \leq \alpha < 1$ is the k-th entry;

Type II:

$$\mathbf{x}_K(\beta) := (1, \dots, 1, \beta) \quad \text{with } \beta \geq (1-q)/q. \tag{17}$$

In terms of the file management policy $\mathbf{p} = (p_0, \dots, p_K) \in (0, q] \times [0, q]^K$, **Type I** threshold policy $\mathbf{x}_k(\alpha)$, uniquely translates into

$$\mathbf{p}_k(\alpha) := (q, 0, \dots, 0, 1-\alpha, q \dots, q, q) \tag{18}$$

where $1-\alpha \in (0, q]$ is the $(k+1)$-st entry $(k = 1, \dots, K)$ (as already observed $p_0 = p_K = q$ in (18) since this is the only solution of the equation $(1-p_K)/p_0 = (1-q)/q$ when $0 \leq p_0, p_K \leq q$ with $p_0 \neq 0$). In particular $\mathbf{p}_1(1-q) = (q, \dots, q)$.

Any file management policy

$$\mathbf{p}_K(\beta) = (p_0, 0, \dots, 0, p_K) \tag{19}$$

with $(1 - p_K)/p_0 := \beta$ corresponds to the unique **Type II** threshold policy $\mathbf{x}_K(\beta)$.

Proposition 2 (Optimality of threshold dynamic policy)

Under the assumption that the utility function $U : \{1, \dots, K\} \to \mathbb{R}^+$ is non-increasing there exists an optimal threshold policy.

Proof. Assume that the optimal policy $\mathbf{x}$ is not a threshold policy. Hence, there exists a k, $1 \leq k \leq K-1$, such that either $x_k < 1$ and $x_{k+1} > 1-q$ if $k \neq K-1$ or $x_{K-1} < 1$ and $x_K > (1-q)/q$ if $k = K-1$.

Assume first that $x_1 \cdots x_{k-1} \neq 0$. Let us show that one can always find $\epsilon_k > 0$ and $\epsilon_{k+1} > 0$ such that $x'_k := x_k + \epsilon_k < 1$, $x'_{k+1} = x_{k+1} - \epsilon_{k+1} > 1-q$ if $k \neq K-1$ (resp. $x'_{k+1} = x_{k+1} - \epsilon_{k+1} > (1-q)/q$ if $k = K-1$) and $G(\mathbf{x}) = G(\mathbf{x}')$, where $\mathbf{x}' = (x_1, \dots, x_{k-1}, x'_k, x'_{k+1}, x_{k+2}, \dots, x_K)$.

Set $\delta_k := x'_k x'_{k+1} - x_k x_{k+1} = \epsilon_k x_{k+1} - \epsilon_{k+1} x_k - \epsilon_k \epsilon_{k+1}$. The identity $G(\mathbf{x}') = G(\mathbf{x})$ is equivalent to

$$x_1 \cdots x_{k-1} (\epsilon_k + \delta_k A_k) = 0$$

that is $\epsilon_k + \delta_k A_k = 0$, with $A_k := 1 + x_{k+2} + x_{k+2}x_{k+3} + \cdots + x_{k+2} \cdots x_K$. The equation $\epsilon_k + \delta_k A_k = 0$ rewrites

$$\epsilon_{k+1} = \epsilon_k \frac{1 + A_k x_{k+1}}{A_k(x_k + \epsilon_k)}.$$

So, we can find ϵ_k and ϵ_{k+1} small enough so that they satisfy the conditions.

Observe that $\epsilon_k + \delta_k A_k = 0$ with $\epsilon_k > 0$ yields $\delta_k < 0$ since $A_k > 0$.

Let us finally show that $F(\mathbf{x}') > F(\mathbf{x})$ which will contradict the optimality of $\mathbf{x}$. We have

$$\begin{aligned}
\frac{F(\mathbf{x}') - F(\mathbf{x})}{x_1 \cdots x_{k-1}} &= \epsilon_k U(k+1) + \delta_k [U(k+2) \\
& \quad + x_{k+2} U(k+3) + \cdots + x_{k+2} \cdots x_{K-1} U(K)] \\
&= (\epsilon_k + \delta_k A_k - \delta_k x_{k+2} \cdots x_K) U(k+1) \\
& \quad + \delta_k [U(k+2) - U(k+1) \\
& \quad + x_{k+2}(U(k+3) - U(k+1)) + \cdots \\
& \quad + x_{k+2} \cdots x_{K-1}(U(K) - U(k+1))] \\
&= -\delta_k x_{k+2} \cdots x_K U(k+1) + \delta_k [U(k+2) \\
& \quad - U(k+1) + x_{k+2}(U(k+3) - U(k+1)) + \cdots \\
& \quad + x_{k+2} \cdots x_{K-1}(U(K) - U(k+1))] \qquad (20)
\end{aligned}$$

where we have used the identity $\epsilon_k + \delta_k A_k = 0$ to derive (20). Since U is non-increasing and $\delta_k < 0$ as noticed earlier, we deduce that the right-hand side of (20) is strictly positive, and therefore $F(\mathbf{x}') > F(\mathbf{x})$.

Assume now that $x_1 \cdots x_{k-1} = 0$. This may only happen when $q = 1$ since $1 - q \leq x_k \leq 1$ for $k = 1, \ldots, K$. Let $l \in \{1, \ldots, k-1\}$ be the smallest integer such that $x_l = 0$.

If the optimal policy is such that $x_l = 0$ then the value of $x_{l+1}, \ldots, x_K$ are irrelevant since $x_l = 0$ implies that $X_{l+1} = \cdots = X_K = 0$ so that both the cost and the constraint will not depend on the values of $x_{l+1}, \ldots, x_K$. Assume for instance that $x_{l+1} = \cdots = x_K = 0$ so that policy $\mathbf{x}$ is of the form $\mathbf{x} = (x_1, \ldots, x_{l-1}, 0, \ldots, 0)$. It this is not a threshold policy then one can find $k' \in \{1, \ldots, l-2\}$ such that $x_{k'} < 1$ and $x_{k'+1} > 1 - q = 0$. We can then duplicate the same argument used to establish (20) with k replaced by k'. Since $x_1 \cdots x_{k'-1} \neq 0$ from the definition of l we conclude that $F(\mathbf{x}') > F(\mathbf{x})$. This completes the proof. ∎

It is actually possible to find the best dynamic file management policy in explicit form, as now shown.

Proposition 3 (Best dynamic file management policy)

Assume that the utility function $U : \{1, \ldots, K\} \to \mathbb{R}^+$ is non-increasing. The following results hold:

- *(a) if $Nq < V$ the optimal file management policy is $\mathbf{p}_1(1-q) = (q, \ldots, q)$;*
- *(b) if $\frac{Nq}{qk+1} < V \leq \frac{Nq}{q(k-1)+1}$ for some $k = 1, \ldots, K$, the optimal file management policy is $\mathbf{p}_k(q(C-k)) = (q, 0 \ldots, 0, 1 - q(C-k), q, \ldots, q)$ (see (18));*
- *(c) if $V \leq \frac{Nq}{q(K-1)+1}$ any file management policy $\mathbf{p}_K(C-K) = (p_0, 0, \ldots, 0, p_K)$ such that $(1 - p_K)/p_0 = C - K$ is optimal.*

Proof. Since we have shown in Proposition 2 that there exists an optimal threshold policy, we only need to focus on threshold policies as defined in (16)-(17). Easy algebra show that

$$G(\mathbf{x}_k(\alpha)) = k + \frac{\alpha}{q}, \quad k = 1, \ldots, K \tag{21}$$

$$G(\mathbf{x}_K(\beta)) = K + \beta, \tag{22}$$

so that $G(\mathbf{x}_1(\alpha_1)) \leq \cdots \leq G(\mathbf{x}_{K-1}(\alpha_{K-1})) \leq G(\mathbf{x}_K(\beta))$ for all $\alpha_1, \ldots, \alpha_{K-1} \in [1-q, 1)$, $\beta \geq (1-q)/q$. From this we deduce that there are three different cases to consider (recall that $C = N/V$):

(a) $C < G(\mathbf{x}_1(1-q)) = 1/q$ or equivalently $V > Nq$;

(b) $G(\mathbf{x}_k(1-q)) \leq C < G(\mathbf{x}_{k+1}(1-q))$ or equivalently $\frac{Nq}{qk+1} < V \leq \frac{Nq}{q(k-1)+1}$;

(c) $C \geq G(\mathbf{x}_K((1-q)/q))$ or equivalently $V \leq \frac{Nq}{q(K-1)+1}$.

Case (a): In this case any threshold policy satisfies the constraint, so that the optimal policy is the policy which maximizes the cost $H(\mathbf{x})$. It is shown in Lemma 3 in the appendix that for each $k = 1, \ldots, K$, the mapping $x_k \to H(\mathbf{x})$ is non-increasing for any $\mathbf{x} = (x_1, \ldots, x_K) \in \mathbf{E}$. Therefore, policy $\mathbf{x}_1(1-q) = (1-q, \ldots, 1-q, (1-q)/q)$ is optimal, or equivalently (see (18)) the file management policy $\mathbf{p}_1(1-q) = (q, \ldots, q)$ is optimal.

Case (b): Assume that $G(\mathbf{x}_k(1-q)) \leq C < G(\mathbf{x}_{k+1}(1-q))$ for some $1 \leq k \leq K-1$. By Lemma 3 in the appendix we see that the best threshold policy is the one which saturates the constraint, namely policy $\mathbf{x}_k(\alpha)$ such that $G(\mathbf{x}_k(\alpha)) = C$, that is $\alpha = q(C-k)$. By (21) this policy is unique and is given by $\mathbf{x}_k(q(C-k))$. Equivalently (see (18)), the optimal file management policy is $\mathbf{p}_k(q(C-k))$.

Case (c): In this case there is no **Type I** policy which satisfies the constraint $G(\mathbf{x}) \geq C$. Among all **Type II** policies satisfying this constraint the one with the smallest K-th entry is the policy such that $G(\mathbf{x}_K(\beta)) = C$, that is (see (19)) policy $\mathbf{x}_K((C-K)) = (1, \ldots, 1, C-K)$. We conclude again from Lemma 3 that this is the optimal policy. Equivalently (see (19)), any file management policy $\mathbf{p}_K(C-K) = (p_0, 0, \ldots, 0, p_K)$ such that $(1-p_K)/p_0 = C-K$ is optimal. This concludes the proof. ∎

Remark 2.3 (Non-uniqueness of best policy in case (c)) *If the constraint is strong in the sense that no* **Type I** *policy can meet it (case (c) of Proposition 3) then any file management policy* $\mathbf{p} = (p_0, 0, \ldots, 0, p_K)$ *such that* $(1-p_K)/p_0 = C-K$ *is optimal, thereby showing that* p_0 *and* p_K *are not unique and, in particular, need not be equal like in cases (a) and (b) of Proposition 3.*

2.3 Numerical results

In all figures the node population is set to 100 ($N = 100$) and the maximum age of copies of F is set to 5 ($K = 5$).

Let $p_s^\star$ (resp. $\mathbf{p}_d^\star$) be the static (resp. dynamic) file management policy which solves the optimization problem $\mathbf{P}$ – as found in Proposition 1 (resp. Proposition 3). Figures 1-4 display the mappings $q \to \sum_{k=1}^{K} U(k)\overline{X}_k$ under both policies $p_s^\star$ and $\mathbf{p}_d^\star$ (corresponding curves are referred to as "static" and "dynamic", respectively), for two different utility functions ($U(k) = 1$, $U(k) = 1/k$) and for two different values of the constraint V ($V = 10, 20$). These results show that the use of the optimal dynamic policy may yield substantial gains (e.g. for $U(k) = 1$ gain of $\approx 22\%$ for all $q \geq 0.2$ – see Fig. 2; gain of $\approx 45\%$ for q close to 1 – see Fig. 1. Gain is halved for $U(k) = 1/k$.). The gain is an increasing function of the meeting probability q.

Figures 5-7 display the mappings $q \to Age(p_s^\star)$ and $q \to Age(\mathbf{p}_d^\star)$ for $V = 20$ and $V = 50$, respectively, where we recall that $Age(\mathbf{p})$ is the expected age of copies of F under policy $\mathbf{p}$ (see (13)). These results show that the best dynamic policy $\mathbf{p}_d^\star$ for *problem* $\mathbf{P}$ may yield a higher expected age than the corresponding optimal static policy $p_s^\star$ (see Figure 6 for $q \geq 0.2$).

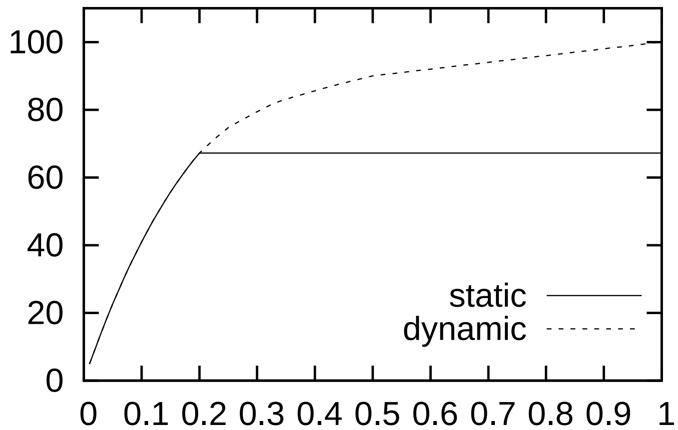

Figure 1: $q \to \sum_{k=1}^{5} \overline{X}_k$ under optimal static/dynamic policy: V=20 (N=100, K=5)

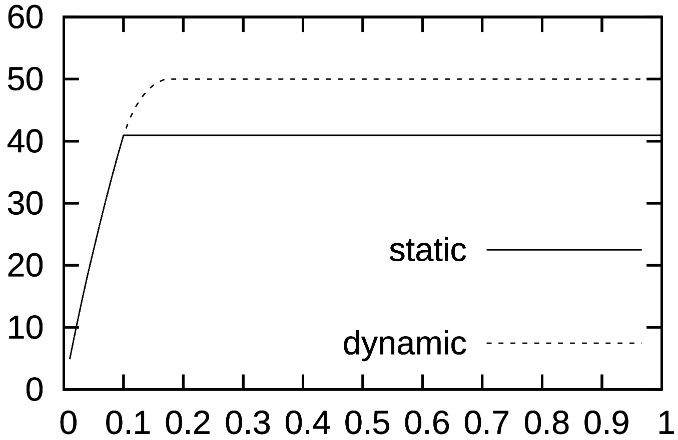

Figure 2: $q \to \sum_{k=1}^{5} \overline{X}_k$ under optimal static/dynamic policy: V=10 (N=100, K=5)

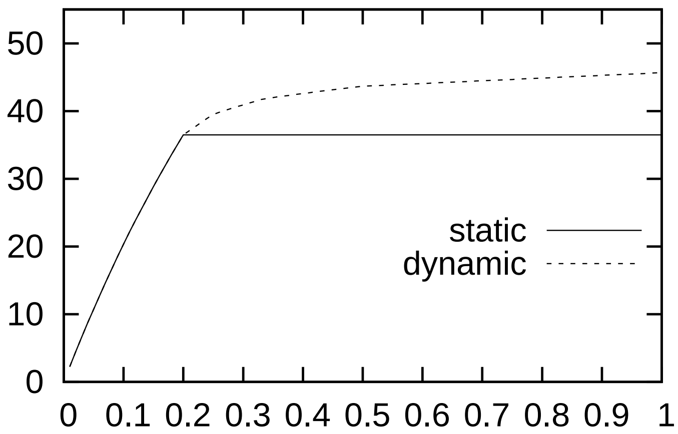

Figure 3: $q \to \sum_{k=1}^{5} \overline{X}_k / k$ under optimal static/dynamic policy: V=20 (N=100, K=5)

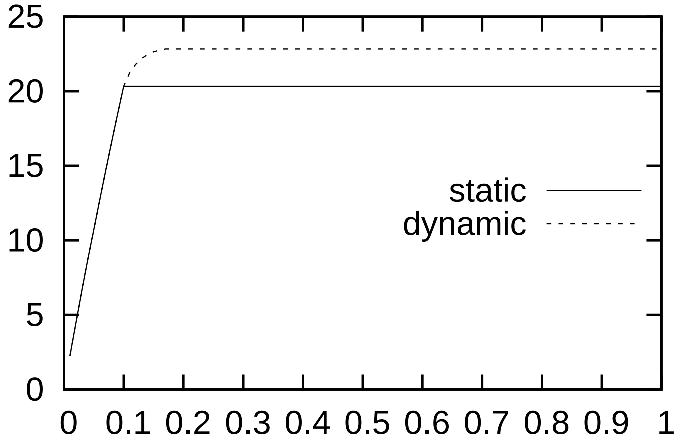

Figure 4: $q \to \sum_{k=1}^{5} \overline{X}_k/k$ under optimal static/dynamic policy: V=10 (N=100, K=5)

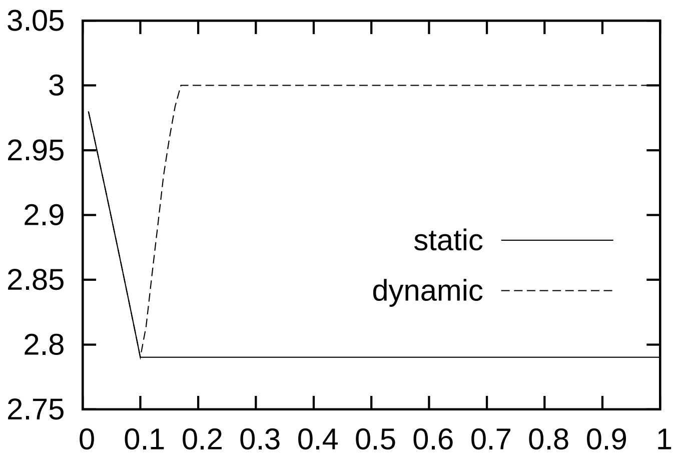

Figure 5: $q \to \{Age(p_s^\star), Age(\mathbf{p}_d^\star)\}$: V=10 (N=100, K=5)

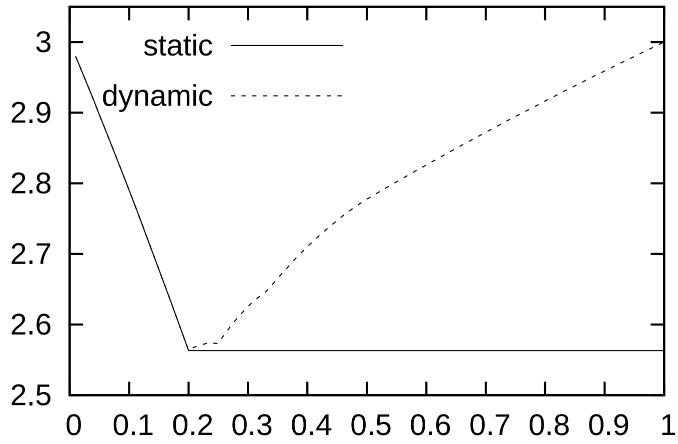

Figure 6: $q \rightarrow \{Age(p_s^\star), Age(\mathbf{p}_d^\star)\}$: V=20 (N=100, K=5)

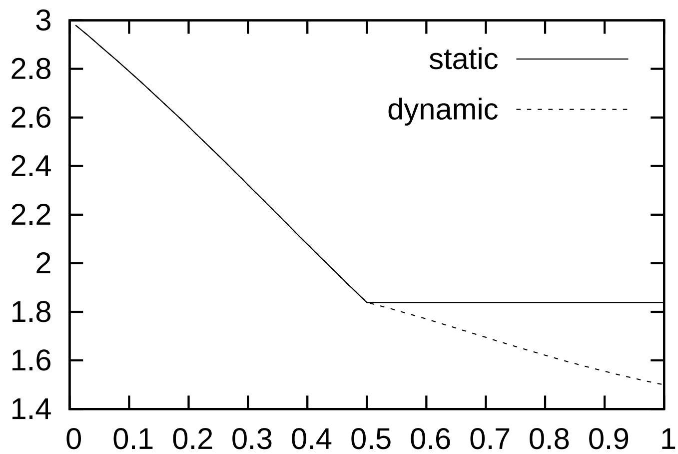

Figure 7: $q \rightarrow \{Age(p_s^\star), Age(\mathbf{p}_d^\star)\}$: V=50 (N=100, K=5)

3 Cooperative nodes

In this section we assume that nodes cooperate in the sense that when two nodes meet the node (including the source) with the most recent version of F may send a copy to the other node. When a node receives a new version of F it deletes at once the version it was carrying, if any. When the source creates a new version of F it deletes at once the previous version.

The identity of the source is 0 and nodes are labeled $1, 2, \ldots, N$. We observe the system at discrete times $\{t_n\}_{n\geq 0}$, where t_n is the time of the nth event. An *event* is either the meeting of the source with a node, the meeting of two nodes or the creation of a new version of F by the source. Let $\{\xi_n^{i,j}\}_n$, $i \neq j$, and $\{\zeta_n\}_n$ be $\{0,1\}$-valued rvs where $\xi_n^{i,j} = 1$, $j \neq 0$, if node i meets node j at time t_n, $\xi_n^{i,0} = 1$ if node i meets the source at time t_n, and $\zeta_n = 1$ if the source creates a new version of F at time t_n. We assume that $\zeta_n + \sum_{i,j} \xi_n^{i,j} = 1$ for all n (only one event at time t_n).

Define $\kappa_n := (\{\xi_n^{i,j}\}), \zeta_n)$ for $n \geq 1$.

Two different scenarii will be consider depending on the amount of information available to the nodes. In scenario I we assume that the nodes know the *age* of the file they carry (if any). More precisely, the age of the version of F carried by node i is equal to $k \geq 1$ if the source has updated file F $k-1$ times since this version was created.

In scenario II we assume that the nodes only know the *date* at which the version of F they carry (if any) has been created by the source.

3.1 Scenario I: Nodes know the age of the file they carry

With a slight abuse of terminology we will say that *node i is of age k* if the version of F that node i it carries is of age k.

Let $Y_n^i \in \{0, 1, 2 \ldots\}$ be the age of node i just before the nth event takes place at time t_n. By convention $Y_n^i = 0$ if node i does not carry any version of F.

We introduce the additional $\{0,1\}$-valued rvs $\{a_n^{i,j}(k,l)\}$ and $\{a_n^i(k)\}$, where $a_n^{i,j}(k,l) = 1$ if node i in state k receives a copy of F from node j in state l if they meet at t_n and $a_n^i(k) = 1$ if the source transmits the latest version of F to node i in state k if they meet at t_n. We assume that $a_n^{i,j}(k,l) = 0$ if $l \geq k$ since clearly a node has no interest to receive a version of F which is either identical or older than the one it carries.

Let $\theta_{i,j}(k,l) := P(a_n^{i,j}(k,l) = 1)$ and $\theta_i(k) := P(a_n^i(k) = 1)$.

The following recursions hold ($i = 1, \ldots, N$):

$$Y_{n+1}^i = Y_n^i + (1 - Y_n^i)\xi_n^{i,0} a_n^i(Y_n^i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} (Y_n^j - Y_n^i)\, \xi_n^{i,j} a_n^{i,j}(Y_n^i, Y_n^j) + \zeta_n. \tag{23}$$

To emphasize the fact that $\theta_{i,j}(k,l) = 0$ if $l \geq k$ we will rewrite (23) as

$$Y_{n+1}^i = Y_n^i + (1 - Y_n^i)\xi_n^{i,0} a_n^i(Y_n^i) + \sum_{\substack{j=1 \\ j \neq i}}^N (Y_n^i - Y_n^j)\, \mathbf{1}_{Y_n^j - Y_n^i < 0}\, \xi_n^{i,j} a_n^{i,j}(Y_n^i, Y_n^j) + \zeta_n. \qquad (24)$$

Define the vectors $Y_n = (Y_n^1, \ldots, Y_n^N) \in \mathcal{E} := \{1, 2, \ldots\}^N$,

Assumptions A1:

(1) $\{\zeta_n\}_n$, $\{\xi_n^{i,j}\}$ and $\{\xi_n^{i,0}\}_n$ are mutually independent renewal sequences with common probability distribution $r := P(\zeta_n = 1)$, $q_{i,j} := P(\xi_n^{i,j} = 1)$ and $q_i := P(\xi_n^{i,0} = 1)$, respectively, for $i, j = 1, \ldots, N$, $i \neq j$;

(2) $r > 0$ and $q_i > 0$ for $i = 1 \ldots, N$;

(3) the probability that two nodes communicate when they meet only depends on their identity and state, namely $P(a_n^{i,j}(Y_n^i, Y_n^j) = 1 | \{Y_m, \kappa_m\}_{m \leq n}) = \theta_{i,j}(Y_n^i, Y_n^j)$ for $i, j = 1, \ldots, N$, $i \neq j$;

(4) given that the source meets a node, the probability that it communicates with it only depends on the node identity and state, that is, $P(a_n^i(Y_n^i) = 1 \,|\, \{Y_m, \kappa_m\}_{m \leq n}) = \theta_i(Y_n^i)$ for $i = 1, \ldots, N$;

(5) If $K = \infty$ then for each $i = 1, \ldots, N$ there exist a finite integer M_i and $\theta_i > 0$ such that $\theta_i(k) \geq \theta_i$ for $k \geq M_i$.

Similar to the non-cooperative setting we define K as the maximum age a version of F can reach. This means that a version of F which reaches age $K + 1$ is deleted at once by the node that carries it. We first study the case where $K = \infty$.

3.1.1 Stability analysis when K is infinite

In this setting the age of a node is unbounded. The result below establishes conditions under which the Markov chain $\{Y_n\}_n$ is stable, that is conditions under which Y_n converges in distribution toward an (a.s.) finite random variable as n goes to infinity.

Proposition 4 (Stability of $\{Y_n\}_n$)

Assume that **A1** *holds.*

$\{Y_n\}_n$ is a time-homogeneous, aperiodic, irreducible and positive recurrent Markov chain on $\mathcal{E}$.

Proof. Only the positive recurrence property does not trivially follow from **A1**(1)-(4). We will show it by applying Foster's criterion (see e.g. [7]) to $\{Y_n\}_n$. Consider the Lyapounov function $f : \mathcal{E} \to \mathbb{R}^+$ defined by $f(\mathbf{y}) = \sum_{i=1}^N y_i$ with $\mathbf{y} = (y_1, \ldots, y_N)$. We need to show that (i) there exist a finite set $\mathcal{F} \subset \mathcal{E}$ and a constant $\epsilon > 0$ such that $E[f(Y_1) - f(\mathbf{y}) \,|\, Y_0 = \mathbf{y}] \leq -\epsilon$ for $\mathbf{y} \in \mathcal{E} - \mathcal{F}$ and (ii) $E[f(Y_1) - f(\mathbf{y}) \,|\, Y_0 = \mathbf{y}] < \infty$ for $\mathbf{y} \in \mathcal{F}$.

Under **A1**(5) define $M_0 := \max\{M_1, \ldots, M_n\} < \infty$ and $\theta := \min(\theta_1, \ldots, \theta_N) > 0$ so that $\theta_i(k) \geq \theta$ for $k \geq M_0$ and $i = 1, \ldots, N$.

Define the Lyapounov function $f(\mathbf{y}) := \sum_{i=1}^N y_i$.

We have from (24)

$$
\begin{aligned}
E[f(Y_1) - f(\mathbf{y}) \,|\, Y_0 = \mathbf{y}] &= \sum_{i=1}^N (1 - y_i) q_i \theta_i(y_i) + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N (y_j - y_i)\, \mathbf{1}_{y_j - y_i < 0}\, q_{i,j} \theta_{i,j}(y_i, y_j) \\
&\quad + Nr \\
&\leq \sum_{i=1}^N (1 - y_i) q_i \theta_i(y_i) + Nr \qquad (25)
\end{aligned}
$$

for $\mathbf{y} \in \mathcal{E}$.

Fix $\epsilon > 0$. Let M be any finite integer such that $M \geq \max\{M_0, (\epsilon + N(r+1))/(\theta \min\{q_1, \ldots, q_N\})\}$. Let $\mathcal{G}$ be a subset of $\mathcal{E}$ defined by $\mathcal{G} := \{\mathbf{y} = (y_1, \ldots, y_N) \in \mathcal{E} \,:\, \max\{y_1, \ldots, y_N\} > M\}$. Fix $\mathbf{y} \in \mathcal{G}$. Let $i^\star$ be such that $y_{i^\star} = \max\{y_1, \ldots, y_N\}$, so that $y_{i^\star} > M$.

From (25) and **A1**(5) we find

$$
\begin{aligned}
E[f(Y_1) - f(\mathbf{y}) \,|\, Y_0 = \mathbf{y}] &\leq \sum_{i=1}^N q_i \theta_i(y_i) - \sum_{i=1, i \neq i^\star}^N y_i q_i \theta_i(y_i) - y_{i^\star} q_{i^\star} \theta_{i^\star}(y_{i^\star}) + Nr \\
&\leq -M \theta q_{i^\star} + N(r+1) \\
&\leq -M \theta \min\{q_1, \ldots, q_N\} + N(r+1) \\
&\leq -\epsilon, \qquad (26)
\end{aligned}
$$

by using the definition of M, which shows part (i) of Foster's criterion.

Define $\mathcal{F} := \mathcal{G}^c = \{\mathbf{y} = (y_1, \ldots, y_N) \in \mathcal{E} \,:\, \max\{y_1, \ldots, y_N\} \leq M\}$. The set $\mathcal{F}$ is a finite and, morever,

$$E[f(Y_1) - f(\mathbf{y}) \,|\, Y_0 = \mathbf{y}] < Nr < \infty, \quad \forall \mathbf{y} \in \mathcal{F},$$

by using (25), which shows part (ii) of Foster's criterion and completes the proof. ∎

We will show in a companion paper that the stability of $\{Y_n\}_n$ can be investigated in a much more general framework than the Markovian framework.

3.1.2 Quantitative performance when K is infinite

We make additional assumptions in order to compute $\overline{X}_k$, the expected number of files of age $k \geq 0$ in steady-state. We assume that the source and node $i = 1, \ldots, N$ (resp. any pair of nodes i and j, $i \neq j$) meet according to a Poisson process with rate $\lambda > 0$ and that the source creates a new version of F at each occurrence of a Poisson process with rate $\mu > 0$. These $N(N+1)/2 + 1$ Poisson processes are assumed to be mutually independent.

We assume that $\theta_i(k) := a_k$ and $\theta_{i,j}(k,l) := b_{k,l}$ for any i, j, k, l. In other words, when two nodes (i.e. source or nodes) meet the probability that a transmission occurs only depends on the nodes state and not on the nodes identity.

Last, we assume that there exist a finite integer M_0 and a constant $\theta > 0$ such that $a_k \geq \theta$ for all $k \geq M_0$, so that the system is stable by Proposition 4 (Hint: apply Proposition 4 with $q_i = q_{i,j} = \lambda/\nu$ and $r = \mu/\nu$ where $\nu := \lambda N(N+1)/2 + \mu$).

Let $X_k(t)$ be number of nodes in state $k \geq 0$ at time t. Set $\overline{X}_k(t) := E[X_k(t)]$.

We have the Kolmogorov equations

$$\begin{aligned}
\frac{d\overline{X}_0(t)}{dt} &= -\lambda a_0 \overline{X}_0(t) - \lambda \sum_{l \geq 1} b_{0,l} E[X_0(t) X_l(t)] && (27)\\
\frac{d\overline{X}_1(t)}{dt} &= -\mu \overline{X}_1(t) + \lambda \sum_{l \geq 0, l \neq 1} a_l \overline{X}_l(t) + \lambda \sum_{l \geq 0, l \neq 1} b_{l,1} E[X_l(t) X_1(t)] && (28)\\
\frac{d\overline{X}_k(t)}{dt} &= \mu \overline{X}_{k-1}(t) + \lambda b_{0,k} E[X_0(t) X_k(t)] + \lambda \sum_{l \geq k+1} b_{l,k} E[X_l(t) X_k(t)] \\
&\quad -\lambda \sum_{l=1}^{k-1} b_{k,l} E[X_k(t) X_l(t)] - (\lambda a_k + \mu) \overline{X}_k(t), \quad k \geq 2. && (29)
\end{aligned}$$

Let $X_k := \lim_{t\to\infty} X_k(t)$ (a.s.) and $\overline{X}_k := \lim_{t\to\infty} E[X_k(t)] = E[X_k]$ where the latter equality comes from the bounded convergence theorem (Hint: $0 \leq X_k(t) \leq N$). From (28)-(29) we find

$$\begin{aligned}
a_0 \overline{X}_0 &= -\sum_{l \geq 1} b_{0,l} E[X_0 X_l] && (30)\\
\mu \overline{X}_1 &= \lambda \sum_{l \geq 0, l \neq 1} a_l \overline{X}_l + \lambda \sum_{l \geq 0, l \neq 1} b_{l,1} E[X_l X_1] && (31)
\end{aligned}$$

$$\mu \overline{X}_{k-1} + \lambda b_{0,k} E[X_0 X_k] + \lambda \sum_{l \geq k+1} b_{l,k} E[X_l X_k] = \lambda \sum_{l=1}^{k-1} b_{k,l} E[X_k X_l] + (\lambda a_k + \mu) \overline{X}_k, \quad k \geq 2. \tag{32}$$

We deduce from (30) that $\overline{X}_0 = b_{0,l}E[X_0X_l] = 0$ for all $l \geq 1$, so that (31)-(32) become

$$\mu\overline{X}_1 = \lambda\sum_{l\geq 2} a_l\overline{X}_l + \lambda\sum_{l\geq 2} b_{l,1}E[X_lX_1] \tag{33}$$

$$\mu\overline{X}_{k-1} + \lambda\sum_{l\geq k+1} b_{l,k}E[X_lX_k] = \lambda\sum_{l=1}^{k-1} b_{k,l}E[X_kX_l] + (\lambda a_k + \mu)\overline{X}_k, \quad k \geq 2. \tag{34}$$

Remark 3.1 *It is not surprising that $\overline{X}_0 = 0$ since, when $K = \infty$, any node will receive a version of F with probability one at some point in time and for this time onwards will never be empty. This implies that $\overline{X}_0 = 0$.*

We will consider two cases.

Case (a): $b_{k,l} = 0$ for all k, l. This case corresponds to the non-cooperative setting studied in Section 2 when $K = \infty$. We find (Hint: use $\sum_{k\geq 1} X_k = N$)

$$\overline{X}_k = \frac{N\prod_{j=2}^{k}\frac{\mu}{\mu+\lambda a_j}}{\sum_{j\geq 1}\prod_{i=2}^{j}\frac{\mu}{\mu+\lambda a_i}}, \quad k \geq 1. \tag{35}$$

If we perform the change of variable $\mu/(\mu + \lambda a_i) = 1 - p_{i-1}$ in (35) we retrieve the corresponding results (8) found in the discrete-time setting with $K = \infty$ (see Remark 2.1), thereby showing that this model is the continuous-time analog of the discrete-time model.

Case (b): $a_k = a > 0$ and $b_{k,l} = b > 0$ for all k, l. Because of the terms $E[X_kX_l]$ equations (31)-(32) cannot be solved. To solve them we will assume that $\mathrm{cov}(X_k, X_l)$ is negligible for $k \neq l$ so that $E[X_kX_l] \approx \overline{X}_k\overline{X}_l$. We conjecture that this approximation (referred to as the "mean-field approximation" – see e.g. [1]) is accurate for large N (the mean-field approach in [6, Theorem 3.1] does not apply here and cannot therefore be used to validate these approximations). With this approximation and the use of the identity $\sum_{k\geq 1}\overline{X}_k = N$, equations (31)-(32) become

$$b\overline{X}_1^2 - \overline{X}_1(bN - a - \mu/\lambda) - aN = 0 \tag{36}$$

$$b\overline{X}_k^2 - \overline{X}_k\left(bN - a - \mu/\lambda - 2b\sum_{l=1}^{k-1}\overline{X}_l\right) + \mu\overline{X}_{k-1}/\lambda = 0 \tag{37}$$

for $k \geq 2$. The unique nonnegative root of (36) is

$$\overline{X}_1 = \left(D_1 + \sqrt{D_1^2 + 4abN}\right)/2b, \tag{38}$$

while for $k \geq 2$ we get from (37)

$$\overline{X}_k = \left(D_k + \sqrt{D_k^2 + 4b\mu \overline{X}_{k-1}/\lambda} \right) /2b \tag{39}$$

with $D_k := bN - a - \mu/\lambda - 2b \sum_{l=1}^{k-1} \overline{X}_l$. Equations (38)-(39) define a recursive scheme allowing the computation of $\overline{X}_k$ for any k.

3.1.3 Quantitative performance when K is finite

In this case the system is always stable since $|Y_n^i| \leq K$ for all $i = 1, \ldots, K$, $n \geq 1$.

When $K < \infty$ the steady-state equations satisfied by $\overline{X}_0, \ldots, \overline{X}_K$ are

$$\mu \overline{X}_K = \lambda a_0 \overline{X}_0 + \lambda \sum_{l=1}^{K} b_{0,l} E[X_0 X_l] \tag{40}$$

$$\lambda a_0 \overline{X}_0 + \lambda \sum_{l=2}^{K} a_l \overline{X}_l + \lambda b_{0,1} E[X_0 X_1] + \lambda \sum_{l=2}^{K} b_{l,1} E[X_l X_1] = \mu \overline{X}_1 \tag{41}$$

$$\mu \overline{X}_{k-1} + \lambda b_{0,k} E[X_0 X_k] + \lambda \sum_{l=k+1}^{K} b_{l,k} E[X_l X_k] = \lambda \sum_{l=1}^{k-1} b_{k,l} E[X_k X_l] + (\lambda a_k + \mu) \overline{X}_k \tag{42}$$

for $k = 2, \ldots, K-1$, and

$$\mu \overline{X}_{K-1} = \lambda \sum_{l=1}^{K-1} b_{K,l} E[X_K X_l] + (\lambda a_K + \mu) \overline{X}_K. \tag{43}$$

Similarly to the setting where $K = \infty$ let us briefly consider the following cases.

Case (a): $b_{k,l} = 0$ for all k, l. This case corresponds to the non-cooperative setting studied in Section 2 when $K < \infty$. Easy algebra yields

$$X_0 = \frac{N \prod_{i=2}^{K} \frac{\mu}{\mu + \lambda a_i}}{R}, \quad X_k = \frac{N(\lambda a_0/\mu) \prod_{i=2}^{k} \frac{\mu}{\mu + \lambda a_i}}{R}, \quad k = 1, \ldots, K$$

where $R := (\lambda a_0/\mu) \sum_{j=1}^{K} \prod_{i=2}^{j} (\mu/(\mu + \lambda a_i)) + \prod_{i=2}^{K} (\mu/(\mu + \lambda a_i))$.

Case (b): $a_k = a > 0$ and $b_{k,l} = b > 0$ for all k, l. Similarly to Case (b) in Section 3.1.2 we assume that $\text{cov}(X_k, X_l) \approx 0$, or equivalently $E[X_k X_l] \approx \overline{X}_k \overline{X}_l$, for all $k \neq l$, since without this assumption it does not seem possible to solve (40)-(43). We conjecture that this assumption is justified when N, the number of nodes, is large.

From (41) we find that $\overline{X}_1$ is given by (38). From (42) we find that $\overline{X}_k$ is given by (39) for $k = 2, \ldots, K-1$. It remains to calculate $\overline{X}_0$ and $\overline{X}_K$. Equation (43) gives

$$\overline{X}_K = \frac{\mu \overline{X}_{K-1}}{\lambda \left(a + b\sum_{l=1}^{K-1} \overline{X}_l\right) + \mu} \tag{44}$$

so that from (40)

$$\overline{X}_0 = \frac{\mu \overline{X}_K}{\lambda \left(a + b\sum_{l=1}^{K} \overline{X}_l\right)}. \tag{45}$$

Remark 3.2 *The case where the rate at which the source meets a node is different from the rate at which two nodes meet is a straightforward extension of the analysis in Sections 3.1.2 and 3.1.3.*

3.1.4 Numerical results

We want to quantify the impact of node cooperation on the system performance when the source has limited power resources. We want to optimize the system utility $\sum_{k=1}^{K} U(k)\overline{X}_k$ under a constraint, denoted by V, on the expected number of transmissions *by the source* between the creation of two consecutive version of F. To this end, we will assume that the source transmits to any node that it meets with the probability $a = a^\star$, where $a^\star := \min(1, (1+\rho)V/N\rho)$ is the static policy that solves problem **P** (Proposition 1). Let q_ρ be the probability that the source meets a given node between two creations of a new version of F. We have $q_\rho = \lambda/(\lambda+\mu) = \rho/(1+\rho)$ thanks to the Poisson assumptions. In all experiments reported below we set $N = 20$, $V = 2$ and $K = 5$. Figures 8-9 display the mapping $q_\rho \to \sum_{k=1}^{K} U(k)\overline{X}_k$ (with $U(k) = 1$ in Fig. 8 and $U(k) = 1/k$ in Fig. 9) for two values of the probability b. The value $b = 0$ corresponds to the non-cooperative setting (case (i); curve referred to as "noncooperative") and the value $b = 0.05$ corresponds to the cooperative setting (case (ii)). For $b = 0.05$ the results have been obtained both by simulations and from the approximation formulas (38)-(39). Note that the approximation developed in Section 3.1.2 works well when $b = 0.05$ (results are not as good as b increases). One observes that the cooperative setting outperforms the noncooperative setting even when the "probability of cooperation" b is small, and that the gain of node cooperation increases with q_ρ.

3.2 Scenario II: Nodes only know the date of creation of their file

The state of a node (including the source) at time t is now defined as the date of creation of the version of F this node is carrying, if any, at time t. More precisely, let $\tilde{Y}_n^i$ be the date of creation (by the source) of the version of F that node $i = 1, \ldots, N$ is carrying just before the nth event takes place, namely at time t_n-. By convention $\tilde{Y}_n^i = 0$ if node i does not

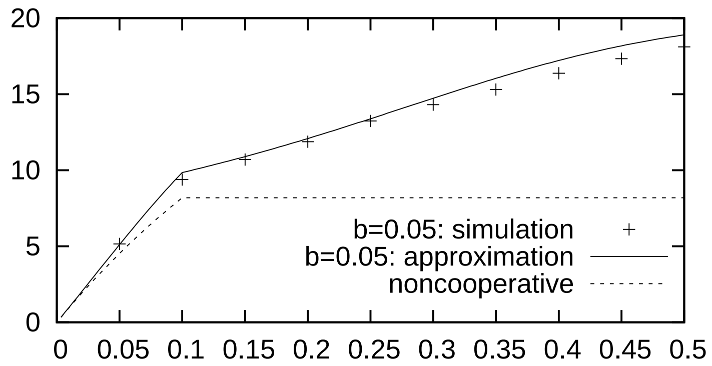

Figure 8: $q_\rho \rightarrow \sum_{k=1}^{5} \overline{X}_k$ ($a = a^\star$, $b \in \{0, 0.05\}$, $N = 20$, $V = 2$, $K = 5$)

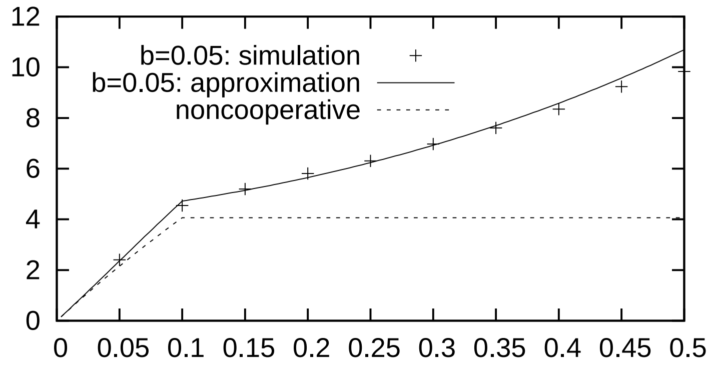

Figure 9: $q_\rho \rightarrow \sum_{k=1}^{5} \overline{X}_k/k$ ($a = a^\star$, $b \in \{0, 0.05\}$, $N = 20$, $V = 2$, $K = 5$)

carry any version of F at time t_n-. Similarly, let $\tilde{Y}_n^0$ be the date of creation of the current version of F that the source is carrying at time t_n-. Without loss of generality we assume that the first version of F is created by the source at time $t = 0$.

Define

$$\tilde{Z}_n^i := \tilde{Y}_n^0 - \tilde{Y}_n^i, \quad i = 0, 1, \ldots, N,\ n = 1, 2, \ldots \tag{46}$$

and let

$$\tilde{Z}_n := (\tilde{Z}_n^1, \ldots, \tilde{Z}_n^N) \in \mathbb{R}_+^N, \quad \tilde{\mathbf{Z}} := \{\tilde{Z}_n\}_n. \tag{47}$$

The rv $\tilde{Z}_n^i$ is the age difference between the most recent version of F and the version of F that node i is carrying (if any) at time t_n-. In particular $\tilde{Z}_n^0 = 0$ since the source always carries the most up to date version of F.

We introduce the $\{0,1\}$-valued rvs $\{\tilde{a}_n^{i,j}(y,z)\}$ where $\tilde{a}_n^{i,j}(y,z) = 1$ if node i in state y receives a file from node j in state z given that they meet at time t_n. We further assume that $\tilde{a}_n^{i,j}(y,z) = 0$ if $y \geq z$, namely a node cannot receive a file from another node that is older than the one it carries.

The following dynamics hold

$$\tilde{Y}_{n+1}^i = \tilde{Y}_n^i + \sum_{\substack{j=0 \\ j \neq i}}^{N} (\tilde{Y}_n^j - \tilde{Y}_n^i)\, \xi_n^{i,j} \tilde{a}_n^{i,j}(\tilde{Y}_n^i, \tilde{Y}_n^j), \quad i = 1, \ldots, N, \tag{48}$$

and

$$\tilde{Y}_{n+1}^0 = \tilde{Y}_n^0 + (t_n - \tilde{Y}_n^0)\zeta_n. \tag{49}$$

Subtracting (48) from (49) and using the definition (46) yields

$$\tilde{Z}_{n+1}^i = \tilde{Z}_n^i + \sum_{\substack{j=0 \\ j \neq i}}^{N} (\tilde{Z}_n^j - \tilde{Z}_n^i)\, \xi_n^{i,j} \tilde{a}_n^{i,j}(\tilde{Y}_n^0 - \tilde{Z}_n^i, \tilde{Y}_n^0 - \tilde{Z}_n^j) + (t_n - \tilde{Y}_n^0)\zeta_n. \tag{50}$$

Assumptions A2:

(1) the successive meeting times between any pair of nodes (including the source) form a Poisson process with rate λ. These $N(N+1)/2$ Poisson processes are mutally independent;

(2) the successive dates of creation by the source of a new version of F follow a Poisson process with rate $\mu > 0$;

(3) the $N(N+1)/2 + 1$ Poisson processes introduced in (1)-(2) above are mutually independent;

(4) for every $n \geq 1$, $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, $i \neq j$, there exists a mapping $\hat{a}_n^{i,j} : \mathbb{R} \to \{0,1\}$ with $\hat{a}_n^{i,j}(x) = 0$ for $x \leq 0$, such that $\tilde{a}_n^{i,j}(y,z) = \hat{a}_n^{i,j}(z-y)$.

(5) for fixed i, j, x, $i \neq j$, $x > 0$, $\{\hat{a}_n^{i,j}(x)\}_n$ is an iid sequence of rvs. The sequences $\{\hat{a}_n^{i,j}(x)\}_n$ and $\{\hat{a}_n^{i',j'}(x')\}_n$ are mutually independent for any $(i, j, x) \neq (i', j', x')$, and for every i, j, x, $i \neq j$, $x > 0$, $\{\hat{a}_n^{i,j}(x)\}_n$ is independent of the $N(N+1)+1$ Poisson processes introduced above in (1)-(2).

Assumption **A2**(4) means that the decision that node i in state y receives a file from node j (including the source) in state $z > y$ only depends on the age difference $z - y$ between the versions of F these two nodes carry.

Corollary 1 *Assumptions A2-(1), A2-(2) and A2-(3) imply that the sequence $\{\xi_n^{i,j}, i = 1, \ldots, N, j = 0, 1 \ldots, N\}_n$ and $\{\zeta_n\}_n$ (defined at the beginning of Section 3) are both iid and mutually independent sequences of rvs with $P(\xi_n^{i,j} = 1) = \lambda/\nu$ and $P(\zeta_n = 1) = \mu/\nu$. The same assumptions together with Assumption A2-(5) imply that for every i, j, x, $i \neq j$, $x > 0$, the rvs $\{\hat{a}_n^{i,j}(x)\}_n$ are independent of the rvs $\{\kappa_n\}_n$.*

Lemma 1 (Markov chain property)

Assume that Assumptions A2 hold. The process $\tilde{\mathbf{Z}}$ defined in (47) is a Markov chain on the state space $\mathbb{R}_+^N$.

Proof. Assumption A2-(4) allows us to rewrite (50) as

$$\tilde{Z}_{n+1}^i = \tilde{Z}_n^i - \sum_{\substack{j=0 \\ j \neq i}}^{N} (\tilde{Z}_n^- - \tilde{Z}_n^j)\, \xi_n^{i,j} \hat{a}_n^{i,j}(\tilde{Z}_n^i - \tilde{Z}_n^j) + (t_n - \tilde{Y}_n^0)\zeta_n. \tag{51}$$

On the event $\{\zeta_n = 0\}$, we readily conclude from (51), Corollary 1 and Assumption A2-(5) that the probability distribution of $\tilde{Z}_{n+1}$ conditioned on the history $\tilde{Z}_1, \ldots, \tilde{Z}_n$ only depends on $\tilde{Z}_n$.

On the event $\{\zeta_n = 1\}$, the same conclusion holds since $(t_n - \tilde{Y}_n^0)\zeta_n$, the second term in the r.h.s. of (51), which is the time that elapses between the creation of the nth and of the $(n+1)$-st version of F by the source (and is therefore distributed as an exponential rv with intensity μ) is independent of $\tilde{Z}_m$ for $m \leq n$ by construction. This completes the proof. ∎

Define the n-step transition probability

$$P^{(n)}(z, A) := P(\tilde{Z}_n \in A \,|\, \tilde{Z}_0 = z), \quad z \in \mathbb{R}_+^N, A \in \sigma(\mathbb{R}_+^N)$$

of the Markov chain $\tilde{\mathbf{Z}}$ and let $P(z, A) := P^{(1)}(z, A)$ be its one-step transition probability.

Let $z_0 := (0, \ldots, 0) \in \mathbb{R}_+^N$ denote the state where all nodes carry the most recent version of F.

Assumption A3:

- For every $i = 1, \ldots, N$ there exist finite constants $0 < \theta_i \le 1$ and $0 < L_i < \infty$ that such $P(\hat{a}_n^{i,0}(z) = 1) = E[\hat{a}_n^{i,0}(z)] \ge \theta_i$ for all $z \ge L_i$.

Assumption **A3** means that there is a non-zero probability that when the source meets a node it transmits F to that node if the copy of F that the node is carrying is old enough w.r.t. to the most recent version of F.

Lemma 2 (ψ-irreducibility of $\{\tilde{Z}_n\}_n$)

Assume that Assumptions **A2**-**A3** *hold.*

The Markov chain $\tilde{\mathbf{Z}}$ *is ψ-irreducible with ψ the maximal irreducibility probability measure on $\sigma(\mathbb{R}_+^N)$ defined by $\psi(A) := \sum_{n\ge1} P(z_0, A)2^{-n}$, $A \in \sigma(\mathbb{R}_+^N)$.*

Proof.

Let us first show that the Markov chain $\tilde{\mathbf{Z}}$ is δ-irreducible with $\delta(\{z_0\}) = 1$ and $\delta(A) = 0$ for all $A \in \sigma(\mathbb{R}_+^N)$ such that $z_0 \notin A$. This amounts to showing that for all $z \in \mathbb{R}_+^N$ there exists $n \ge 1$ such that $P^n(z, z_0) > 0$ [7, p. 87].

Consider the following chain of events: at time $t = 0$ the source creates a new version of F and this version is created at least $L := \max\{L_1, \ldots, L_N\}$ units of time after the previous creation and at times $0 < t_1 < \ldots < t_N$ the source meets nodes $1, \ldots, N$, respectively and transmits the latter version of F to each node. The probability of occurence of this chain of events is strictly positive under Assumptions **A2**-**A3**.

Conditioned on these $N + 1$ events we observe from the definition of the Markov chain $\tilde{\mathbf{Z}}$ that $\tilde{Z}_{N+1} = z_0$ (i.e. just before time t_{N+1} all nodes carry the most up to date version of F) regardless of the value of the chain at time $t = 0$. This shows that $P^{N+1}(z, z_0) > 0$ for all $z \in \mathbb{R}_+^N$ and therefore that $\tilde{\mathbf{Z}}$ is δ-irreducible.

On $\sigma(\mathbb{R}_+^N)$ define the probability measure $\psi(A) = \sum_{n\ge1} P^n(z_0, A)2^{-n}$, $A \in \sigma(\mathbb{R}_+^N)$. Assume that $\psi(A) > 0$ for $A \in \sigma(\mathbb{R}_+^N)$. Hence, there exists $n \ge 1$ such that $P^n(z_0, A) > 0$. From the inequality

$$P^{N+1+n}(z, A) \ge P^{N+1}(z, z_0)\, P^n(z_0, A)$$

and the property that $P^{N+1}(z, z_0) > 0$ as shown above, we conclude that $P^{N+1+n}(z, A) > 0$, so that the Markov chain $\tilde{\mathbf{Z}}$ is ψ-irreducible.

The fact that ψ is a maximal irreducibility probability measure follows from [7, Prop. 4.2.2] (see also [7, Prop. 4.3.1] for a related case study). ∎

Proposition 5 (Stability of $\{\tilde{Z}_n\}_n$) *Assume that Assumptions* **A2**-**A3** *hold. Then, the Markov chain* $\tilde{\mathbf{Z}}$ *is positive Harris recurrent.*

Proof. Let us show that the conditions of Theorem 11.3.4 in [7, p. 265] hold.

We have already shown in Lemma 2 that $\tilde{\mathbf{Z}}$ is ψ-irreducible with ψ the maximal irreducibility probability measure on $\sigma(\mathbb{R}_+^N)$. Theorem 11.3.4 in [7, p. 265] will apply if one can find a petite set C [7, p. 121], a function $V : \mathbb{R}_+^N \to [0, \infty]$ bounded on C and some constant $b < \infty$ such that

$$E[V(\tilde{Z}_1) \,|\, \tilde{Z}_0 = z] - V(z) \le -1 + b\mathbf{1}_C(z), \quad z = (z_1, \ldots, z_N) \in \mathbb{R}_+^N. \tag{52}$$

Consider the function $V : \mathbb{R}_+^N \to [0, \infty]$ defined by $V(z) = \sum_{i=1}^N z_i$ with $z = (z_1, \ldots, z_N)$.

Let L be any integer such that

$$L > \max\left\{L_1, \ldots, L_N, \frac{\nu + N}{\lambda \min_{1\le i\le N} \theta_i}\right\}$$

where we recall that $\nu = \lambda N(N+1)/2 + \mu$ and that constants L_i's are defined in Assumption **A3**. Let C be the bounded set defined by

$$C = \left\{z = (z_1, \ldots, z_N) \in \mathbb{R}_+^N \;:\; \sum_{i=1}^N z_i \le LN\right\}. \tag{53}$$

Observe that V is bounded on C.

Fix $z \in \mathbb{R}_+^N - C$ and let $i^\star$ be such that $z_{i^\star} > L$ (Hint: $\sum_{n=1}^N z_n > LN$ implies that at least one z_i is larger than L).

We have from (51) (Hint: $a_n^{i,j}(z) = 0$ if $z \le 0$ by Assumption **A2** and $\tilde{Z}_n^0 = 0$ by construction)

$$\begin{aligned}
E[V(\tilde{Z}_1) \,|\, \tilde{Z}_0 = z] - V(z) &\le -E\left[\sum_{i=1}^N \sum_{j=1}^N (z_i - z_j)\xi_n^{i,j} \hat{a}_n^{i,j}(z_i - z_j)\right] \\
&\quad -E\left[\sum_{i=1}^N z_i \xi_n^{i,0} \hat{a}_n^{i,0}(z_i)\right] + NE[(t_n - Y_n^0)\zeta_n] \\
&\le -\frac{\lambda}{\nu}\sum_{i=1}^N z_i E[\hat{a}_n^{i,0}(z_i)] + NE[(t_n - Y_n^0)\zeta_n] \quad \text{since } E[\xi_n^{i,0}] = \lambda/\nu \text{ by Corollary 1} \\
&\le -\frac{\lambda}{\nu} z_{i^\star} E[\hat{a}_n^{i^\star,0}(z_{i^\star})] + \frac{N}{\nu} \quad \text{since } E[(t_n - Y_n^0)\zeta_n] = \nu \\
&\le -\frac{L\lambda}{\nu}\theta_{i^\star} + \frac{N}{\nu} \quad \text{from Assumption } \mathbf{A3} \text{ and } z_{i^\star} > L_{i^\star} \\
&\le -\frac{L\lambda}{\nu} \min_{1\le i\le N}\{\theta_i\} + \frac{N}{\nu} \\
&\le -1
\end{aligned}$$

from the definition of L. On the other hand, for $z \in C$ we have $E[V(\tilde{Z}_1) \,|\, \tilde{Z}_0 = z] - V(z) \le N/\nu$ (this inequality actually holds for all $z \in \mathbb{R}_+^N$).

In summary, we have shown that (52) holds for the Lyapounov function $V(z) = \sum_{i=1}^{N} z_i$, the finite set C defined in (53), and the constant $b = N/\nu + 1$. It remains to check that the set C is petite.

We have shown in the proof of Lemma 2 that there exists a constant $\omega > 0$ such that $P^{N+1}(z, z_0) > \omega$ for all $z \in \mathbb{R}_+^N$. Hence, by the Chapman-Kolgomorov equations, we have that

$$P^{N+2}(z, A) \geq P^{N+1}(z, z_0)\, P(z_0, A) \geq \omega P(z_0, A) \tag{54}$$

for all $z \in C$, $A \in \sigma(\mathbb{R}_+^N)$. Since $\omega P(z_0, \cdot)$ is a non-trivial measure on $\sigma(\mathbb{R}_+^N)$ (54) shows that the set C is small [7, p. 106], and thereby that C is petite with the sampling distribution taken as the Dirac measure at point $\{N + 2\}$ – see comment after the definition of a petite set in [7, p. 121]. This concludes the proof. ∎

4 Imperfect state information

In this section we consider the static setting of Section 2 where nodes do not cooperate. We assume that the source does not know parameters N and q, so that it cannot compute $a^\star := \min(1, V/Nq)$, the (conditional) transmission probability that solves problem **P** (cf. Proposition 1). Instead, we will assume that every $M \geq 1$ slots the source updates the transmission probability a, where M is an arbitrary integer. More precisely, let θ_m be the transmission probability used in slots $mM, \ldots, (m+1)M-1$. Define the projection operator

$$\Pi_H(u) = \begin{cases} 1 & \text{if } u > 1 \\ u & \text{if } 0 \leq u \leq 1 \\ 0 & \text{if } u < 0. \end{cases}$$

Consider the stochastic recursion

$$\theta_{m+1} = \Pi_H\Big(\theta_m + \epsilon_m(MV - Y_m)\Big) \tag{55}$$

where Y_m is the total number of transmissions in slots $mM, \ldots, (m+1)M - 1$, and $\{\epsilon_m\}_m$ are nonnegative real numbers satisfying

$$\sum_{m\geq 0} \epsilon_m^2 < \infty, \quad \sum_{m\geq 0} \epsilon_m = \infty. \tag{56}$$

Observe that the source knows Y_m for every m. Recursion (55) is motivated by the fact that $a^\star$ is the unique zero of $h(a) := V - \overline{X}_1$ if $h(1) > 0$ and $a^\star = 1$ otherwise, so that the source target is to find the zero, if any, of $h(a)$ (or, equivalently, the zero of $Mh(a)$) in [0,1].

Proposition 6 (Stochastic approximation algorithm)

As $m \to \infty$, θ_m in (55) converges with probability one to $a^\star$, the optimal static policy of Section 2.2.1.

Proof. The proof directly follows from the remark after Theorem 2.1 in [8, p. 127]. Let us briefly checked that conditions (A2.1)-(A2.5) of Theorem 2.1 hold. Since $0 \leq Y_m \leq MN$ for all m, condition (A2.1) holds (this condition requires that $\sup_m E|Y_m|^2 < \infty$). By an inductive argument applied to (55) we see that $E[Y_m|\theta_0, Y_i, i < m] = E[Y_m|\theta_m, \theta_i, Y_i, i < m]$. We then note that $E[Y_m|\theta_m, \theta_i, Y_i, i < m] = E[Y_m|\theta_m] := g(\theta_m)$ since the decision by the source to transmit a copy of F to a node only depends on the enforced transmission probability. This implies that condition (A2.2) holds (condition (A2.2) in [8, p. 126] states that $E[Y_m|\theta_0, Y_i, i < m]$ has the form of $g(\theta_m) + \beta_m$ where β_n is a r.v.). We have

$$g(x) = M(V - Nqx)$$

so that conditions (A2.3) (g is continuous) and (A2.5) ($\sum_{m\geq 0} \epsilon_m |\beta_m| < \infty$ w.p.1) are satisfied. Last, condition (A2.4) ($\sum_{m\geq 0} \epsilon_n^2 < \infty$) holds from (56).

Consider the ODE $dx(t)/dt = g(x(t))$. Its solution is $x(t) = (x(0) - V/Nq)e^{-MNqt} + V/Nq$. It has a unique equilibrium point, given by $x_0 = V/Nq$, which is asymptotically stable in the sense of Lyapounov [8, p. 104] (i.e. for each $\delta > 0$, there exists $\eta > 0$ such that if $|x(0) - x_0| < \eta$ then $|x(t) - x_0| < \epsilon$ for all $t \geq 0$). By [8, Remark p. 127] we conclude that $\{\theta_m\}_m$ converges with probability one to $\min(1, V/Nq)$. ∎

Figure 10 below provides a numerical illustration of the convergence of algorithm (55) to the optimal policy $a^\star$ for $M = 1$, $N = 100$, $V = 10$ and $q = 0.2$. In this case $a^\star = 0.5$.

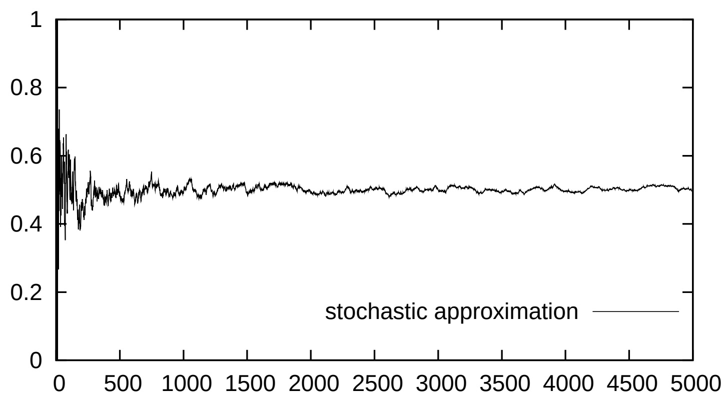

Figure 10: $m \to \theta_m$: $M = 1$, $a^\star = 0.5$ ($N = 100$, $V = 10$, $q = 0.2$)

5 Conclusion

We have developed simple stochastic models for evaluating the performance of file management policies in DTNs storing dynamic files. Both static and dynamic policies have been investigated. We have shown that using dynamic policies instead of static policies yields substantial gain in the performance; this result holds both in the non-cooperative setting, where only the source is allowed to communicate with the other nodes, and in the cooperative setting where all pairwise communications are possible. Future works include the study of multi-source and multi-file scenarii.

6 Acknowlegements

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Lemma 3 (Monotonicity of $H(\mathbf{x})$)

For each $k = 1, \ldots, K$, the mapping $x_k \to H(\mathbf{x})$ is non-increasing for any $\mathbf{x} = (x_1, \ldots, x_K) \in$ $\mathbf{E}$.

Proof. First, notice that the mapping $x_K \to H(\mathbf{x})$ is clearly non-increasing since x_K only appears in $G(\mathbf{x})$, the denominator of $H(\mathbf{x})$, and since $G(\mathbf{x})$ is non-decreasing in x_K.

Assume now that $k = 1, \ldots, K$. Let

$$
\begin{aligned}
B(j) &:= 1 + x_1 + x_1 x_2 + \cdots + x_1 \cdots x_j \\
B_k(j) &:= 1 + x_{k+1} + x_{k+1} x_{k+2} + \cdots + x_{k+1} \cdots x_j
\end{aligned}
$$

with $B(0) = 1$, $B_k(k) = 1$. Set $U(K+1) = 0$. We have

$$
\begin{aligned}
F(\mathbf{x}) &= \sum_{j=1}^{K} [U(j) - U(j+1)] B(j-1) \\
G(\mathbf{x}) &= B(K)
\end{aligned}
$$

so that

$$
\begin{aligned}
\frac{\partial}{\partial x_k} F(\mathbf{x}) &= \prod_{j=1}^{k-1} x_j \sum_{j=k+1}^{K} [U(j) - U(j+1)] B_k(j-1) \\
\frac{\partial}{\partial x_k} G(\mathbf{x}) &= B_k(K) \prod_{j=1}^{k-1} x_j .
\end{aligned}
$$

Therefore

$$\frac{\partial}{\partial x_k} H(\mathbf{x}) = \frac{(\prod_{j=1}^{k-1} x_j)^2}{G(\mathbf{x})^2} \left(\sum_{j=1}^{k} [U(j+1) - U(j)] \times B(j-1) B_k(K) + \sum_{j=k+1}^{K} [U(j+1) - U(j)] \right.$$
$$\left. \times [B(j-1) B_k(K) - B_k(j-1) B(K)] \right).$$

The first summation is non-positive since U is non-increasing and since $B(j-1)B_k(K) \geq 0$ for all $\mathbf{x} \in \mathbf{E}$. Using again the decreasingness of U a sufficient condition for the second summation to be non-positive is that coefficients $B(j-1)B_k(K) - B_k(j-1)B(K)$ are all non-negative. To see that this is indeed true, note that $B(j) = B(k-1) + x_1 \ldots, x_k B_k(j)$ so that $B(j-1)B_k(K) - B_k(j-1)B(K) = B(k-1)[B_k(K) - B_k(j-1)]$ which is non-negative for all $\mathbf{x} \in \mathbf{E}$. This concludes the proof. ∎

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